

**Home work**

**6.1,6.2**



**November 8, 2020**

**Raihan md rakibul islam**

**2020380029**

**#6.1**

4. A particular brand of shirt comes in 12 colors, has a male version and a female version, and comes in three sizes for each sex. How many different types of this shirt are made?

**Color=12**

**Gender=2**

**Size=3**

**We need to use the product rule, because the first event is picking the color, the second event is picking the gender and the third event is picking the size.**

**12\*2\*3 = 72**

12. How many bit strings are there of length six or less, not counting the empty string?

**+ = 64+32+16+8+4+2 = 126**

16. How many strings are there of four lowercase letters that have the letter x in them?

There are 26 possible letters in the alphabet

**Strings of length 4 We need to use the product rule, because the first event is picking the first bit, the second event is picking the second bit, .., the 4th event is picking the 4th bit.**

**26 \*26 \*26 \*26= = 456, 976**

**Strings of length 4 without an x We need to use the product rule, because the first event is picking the first bit, the second event is picking the second bit, .., the 4th event is picking the 4th bit.**

**Note: When the string cannot contain an x, then there are 25 possible letters**

**25 \* 25 \* 25 \* 25 = = 390, 625**

**Strings of length 4 with at least one x Strings of length 4 with at least one x are strings of length 4 that are not strings of length 4 without an x**

**456, 976 - 390, 625 = 66, 351**

30. How many license plates can be made using either three uppercase English letters followed by three digits or four uppercase English letters followed by two digits?

**Three letters followed by three digits**

**There are 26 possible letters and 10 possible digits.**

**First letter= 26 ways**

**Second letter= 26 ways**

**Third letter= 26 ways**

**First digit= 10 ways**

**Second digit= 10 ways**

**Third digit= 10 ways**

**Using the product rule: 26 · 26 · 26 · 10 · 10 · 10 = . = 17, 576, 000**

**Four letters followed by two digits**

**There are 26 possible letters and 10 possible digits.**

**First letter= 26 ways**

**Second letter= 26 ways**

**Third digit= 26 ways**

**Fourth digit= 26 ways**

**First digit= 10 ways**

**Second digit= 10 ways**

**Using the product rule: 26 · 26 · 26 · 26 · 10 · 10 = \* = 45, 697, 600**

**Three letters followed by three digits or Four letters followed by two digits**

**Use the sum rule: 17, 576, 000 +45, 697, 600 = 63, 273, 600**

**#6.2**

8. Show that if f is a function from S to T, where S and T are finite sets with |S| > |T |, then there are elements s1 and s2 in S such that f (s1) = f (s2), or in other words, f is not one-to-one.

**Given: f:S🡪 T**

**S and T are finite sets with |S| > |T|**

**To proof: f is one-to-one**

**PROOF BY CONTRADICTION**

**Let us assume, for the sake of contradiction, that f is one to one.**

**Let n and m be positive integers such that |S| = n and** **|T| = m. Note that these integers need to exist as S and T are finite sets.**

**n<m**

**Let S = {s1,., Sn} and T = {t1, ., tm}.**

**Without loss of generality, we can assume that f(s*i*) = t*i* for *i* = 1,2, ., n as each element needs to have a unique image (since f is one-to-one) and we can rename the elements t1,. tm if the order of the elements didn't match up with the images.**

**Since m > n, we have a term while we do not have a term . This then implies that needs to be the image of some s (with k a positive integer from 1 to n).**

**= f(S*k*) = t*k***

**However, = f(S*k*) = t*k* is then in contradiction with the fact that f is one-to-one. Since we derive a contradiction, our assumption that f is one-to-one is incorrect and thus f is not one-to-one.**

12. How many ordered pairs of integers (a, b) are needed to guarantee that there are two ordered pairs (a1, b1) and (a2, b2) such that a1 mod 5 = a2 mod 5 and b1 mod 5 = b2 mod 5?

**In general, r mod 5 can take on the values 0, 1, 2,3 or 4.**

**The possible combinations for points modulo 5 are then: (0,0) (0,1) (0,2) (0,3) (0, 4) (1,0) (1,1) (1,2) (1,3) (1,4) (2,0) (2, 1) (2, 2) (2,3) (2, 4) (3,0) (3, 1) (3,2) (3, 3) (3, 4) (4,0) (4, 1) (4, 2) (4,3) (4, 4)**

**We then note that there are 25 different points modulo 5 (5 options for the first coordinate, 5 options for the second coordinate, thus in total 5\*5 = 25 combinations).**

**By the Pigeonhole principle, we then know that at least two points have the same coordinates modulo 5 when there are 25 +1 = 26 points.**

26. Show that in a group of five people (where any two people are either friends or enemies), there are not necessarily three mutual friends or three mutual enemies.

**Let us fix a person from the group of five people, say A. Consider the case that out of the remaining four, exactly two are friends of A and exactly two are enemies. If D is a friend to B while an enemy to C. and E is exactly the opposite of D, then we can never have three mutual friends or enemies from them too.**